



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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9231/22

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

1 It is given that $y = \sinh(x^2) + \cosh(x^2)$.

- (a)** Use standard results from the list of formulae (MF19) to find the Maclaurin's series for y in terms of x up to and including the term in x^4 . [2]

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- (b)** Deduce the value of $\frac{d^4y}{dx^4}$ when $x = 0$. [1]

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- (c)** Use your answer to part **(a)** to find an approximation to $\int_0^{\frac{1}{2}} y \, dx$, giving your answer as a rational fraction in its lowest terms. [2]

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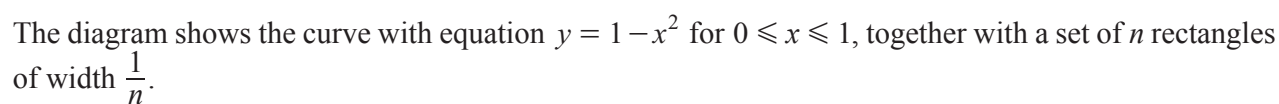
2 Find the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x^3 y}{x^4 + 5} = 6x$$

for which $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$.

[7]

[illegible]


$$\int_0^1 (1-x^2) dx < \frac{4n^2+3n-1}{6n^2}. \quad [4]$$
[illegible]

- (b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 (1-x^2) dx$. [4]

- 4 (a) Write down all the roots of the equation $x^5 - 1 = 0$. [2]

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- (b) Use de Moivre's theorem to show that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$. [4]

[illegible]

$$8x^9 - 8x^7 + x^5 - 8x^4 + 8x^2 - 1 = 0$$

in the form $\cos k\pi$, where k is a rational number. [4]

[illegible]

5 The curve C has parametric equations

$$x = 3t + 2t^{-1} + at^3, \quad y = 4t - \frac{3}{2}t^{-1} + bt^3, \quad \text{for } 1 \leq t \leq 2,$$

where a and b are constants.

(a) It is given that $a = \frac{2}{3}$ and $b = -\frac{1}{2}$.

Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \frac{25}{4}(t^2 + t^{-2})^2$ and find the exact length of C . [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** It is given instead that $a = b = 0$.

Find the value of $\frac{d^2y}{dx^2}$ when $t = 1$.

[4]

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

6 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 6 & 6 \\ 0 & 2 & 6 \\ 0 & 0 & -3 \end{pmatrix}.$$

(a) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [5]

[illegible]

(b) Find the matrix $\mathbf{A}$ such that

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{pmatrix}. \quad [4]$$

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(c) State the eigenvalues and corresponding eigenvectors of $\mathbf{A}^3$. [2]

[illegible]

7 It is given that $y = x^2w$ and

$$x^2 \frac{d^2 w}{dx^2} + 4x(x+1) \frac{dw}{dx} + (5x^2 + 8x + 2)w = 5x^2 + 4x + 2.$$

(a) Show that

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 5x^2 + 4x + 2. \quad [4]$$

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[Turn over

- 8 (a) Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$1 - \tanh^2 x = \operatorname{sech}^2 x. \quad [3]$$

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- (b) Using the substitution $u = \tanh x$, or otherwise, find $\int \operatorname{sech}^2 x \tanh^2 x \, dx$. [2]

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It is given that, for $n \geq 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \tanh^2 x \, dx$.

- (c) Show that, for $n \geq 2$,

$$(n+1)I_n = \left(\frac{4}{5}\right)^3 \left(\frac{3}{5}\right)^{n-2} + (n-2)I_{n-2}. \quad [5]$$

[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$.]

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(d) Find the value of I_4 .

[3]

[illegible]

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